



## MCAMC 2021 Answers

### Individual Round

- |                                       |                                       |
|---------------------------------------|---------------------------------------|
| 1. 70                                 | 11. 3                                 |
| 2. 7                                  | 12. $39 \left(\frac{23}{16}\right)$   |
| 3. 9                                  | 13. $109 \left(\frac{1}{108}\right)$  |
| 4. 120                                | 14. 15048                             |
| 5. $2597 \left(\frac{5}{2592}\right)$ | 15. 8 (2,-2)                          |
| 6. 16                                 | 16. $227 \left(\frac{99}{128}\right)$ |
| 7. 48                                 | 17. 4041                              |
| 8. 50                                 | 18. 8640                              |
| 9. 3                                  | 19. $5 \left(\frac{3}{2}\right)$      |
| 10. 35                                | 20. 5                                 |

## Team Round

1. (6, 2, 7)
2. High price for both firms
3. None
4. Low price for both firms
5. \$0
6. 3
7.  $x \geq 3$
8. Both players write down \$5
9. There is no dominant strategy
10. 30%
11. 48:0:1:0:1 from oldest to youngest
12. 47:0:1:0:2 from oldest to youngest
13. The oldest academy student can give the award to anyone except themselves or the second oldest student
14. Yes, remove 3 matchsticks and then remove what is remaining after player B makes their move
15. 1,237,236; 1,237,240; 1,237,244; 1,237,248; 1,237,252; 1,237,256; 1,237,260; 1,237,264
16. Player 2 should put each matchstick in its own pile
17. Player A does have a winning strategy: remove one stick from the left pile and then match whatever player B does for the opposite pile
18. (1,237,233 , 1,237,233) (1,237,234 , 1,237,234) (1,237,235 , 1,237,235) (1,237,236 , 1,237,236) (1,237,237 , 1,237,237) (1,237,238 , 1,237,238) (1,237,239 , 1,237,239) (1,237,240 , 1,237,240)
19. Alice picks the number 25. For whatever number Bob chooses, she picks 50 minus that number
20. 6

## Live Round

1. 1122
2. 2
3. 28
4. 53
5. 120
6. 64
7. 7
8.  $30(25\sqrt{3} - \frac{\pi}{2})$
9. 14
10.  $889(\frac{160}{729})$
11. 0
12.  $13(\frac{11}{2})$
13. 10010 or 10001100010
14.  $7(\frac{4}{3})$
15. 247
16.  $363(\frac{280}{83})$
17. 128
18. 108
19.  $12(\frac{1}{2})$
20.  $43(2\sqrt{41})$
21. 16
22.  $65(\frac{32}{33})$
23.  $1101(\frac{1001}{100})$
24.  $10(\frac{3\pi}{4} - \frac{1}{2})$