# 1 Live Round: Set 1

1. Harry needs your help evaluating this expression:

$$111 * 10 + 11 + 1.$$

   Can you help him?

2. Circle 1 is inscribed inside of square 1, and square 2 is inscribed in circle 1. What is the ratio between the areas of square 1 and square 2?

3. Professor Trelawney is teaching the Gryffindors about planetary alignments, and says that if two planets can be connected by a line drawn from the Sun, they are in conjunction. Conjunctions can happen between 2 or more planets, but here, we consider conjunctions of exactly 2 planets. Given that there are 8 planets, how many different conjunctions can there be (Location of the conjunction does not matter)?

# 2 Live Round: Set 2

1. Luna and Hermione have been punished for talking during Arithmancy class. They are being required to cut the grass on the Quidditch field with scissors. By herself Luna would take 5 days to finish cutting the entire field while it would take Hermione 4 days to cut the grass. How many hours does it take for them to cut it when they work together? (Round to the nearest hour.)

2. There was a large debate concerning the order of the obstacles guarding the Sorcerer's Stone in Harry's first year. There were seven different obstacles. There were two rules, however:

   (a) Hagrid's obstacle (one of the obstacles) had to be first.

   (b) Snape's obstacle (another one of the obstacles) had to be last.

   How many possible orders were there for the obstacles?

3. A composite number $n$ is known to be magical if the number of its positive divisors is divisible by 7. What is the least positive magical number?

# 3 Live Round: Set 3

1. Gilbert the Gecko lives on (1, 3) on the coordinate plane. His house is part of a square neighborhood with vertices (0,0), (4, 0), (4, 4), and (0, 4). Everyday, he travels either up, down, left, or right randomly with equal probability by 1 unit. The probability that, after at most 3 moves, Gilbert leaves his neighborhood (goes outside the square) is $\frac{m}{n}$, where m and n are relatively prime integers. Find m+n. Note: Once he leaves the neighborhood he does not move anymore. Being on the border is also leaving the neighborhood.

2. Three shaded circles of radius 1 are centered on the vertices of an equilateral triangle of side length 10. The area of the triangle outside the circles (the area of the non-shaded region of the triangle) can be written as $a\sqrt{b} - \frac{\pi}{c}$, where $a, c$ are positive integers and $b$ is a square-free positive integer. Find $a + b + c$.

3. Find the number of real solutions to $7\cos(\pi x) = |x|$

# 4 Live Round: Set 4

1. Ron has three Galleons in his pocket. Two of the Galleons are fair, and one Galleon is double-headed. For 6 iterations, he picks a Galleon at random from his pocket and flips it and then proceeds to put it back in his pocket. The probability that he gets exactly 3 heads out of the 6 flips is $\frac{m}{n}$, where m and n are relatively prime integers. Find $m + n$.

2. Find all real values of x satisfying: $2^x$=$8^{3x} \cdot 64^{2x}$

3. There is a circle centered about point $A$. A radius is drawn from point $A$ to point $D$, a point anywhere on the circle. Chord $BC$ intersects $AD$ at point $E$. $BE = 2$, $EC = 5$. If $ED = 1$, the radius of the circle can be written as $\frac{m}{n}$, where m and n are relatively prime integers. Find $m + n$.

# 5 Live Round: Set 5

1. Harry needs your help evaluating this expression in binary:

$$111 * 10 + 11 + 1.$$

Can you help him find the answer in binary?

2. Regular hexagon 1 is inscribed inside of a circle. Circle 1 is inscribed inside Hexagon 1. Regular hexagon 2 is inscribed inside of Circle 1. The ratio of the area of Hexagon 1 to Hexagon 2 can be written as $\frac{m}{n}$, where m and n are relatively prime integers. Find $m + n$.

3. Professor Trelawney is teaching the Gryffindors about planetary alignments, and says that if two planets can be connected by a line drawn from the Sun, they are in conjunction. Conjunctions can happen between 2 or more planets, but here, we consider conjunctions of any number of planets greater than 1. Given that there are 8 planets, how many different conjunctions can there be? (Location of the conjunction does not matter)

# 6 Live Round: Set 6

1. Luna, Ron, and Hermione have been punished for talking during Arithmancy class. They are being required to cut the grass on the Quidditch field with scissors. Together, Luna and Ron would take 5 days to finish cutting the entire field, Ron and Hermione would take 7 days, and Luna and Hermione would take 4 days to cut the grass. The number of days it takes for them to cut it when all 3 of them work together can be written as $\frac{m}{n}$, where m and n are relatively prime integers. Find $m+n$.

2. There was a large debate concerning the order of the obstacles guarding the Sorcerer's Stone in Harry's first year. There were seven different obstacles. There were three rules, however:

   (a) Hagrid's obstacle (one of the obstacles) has to be placed directly before Professor Flitwick's obstacle.

   (b) Snape's obstacle (another one of the obstacles) has to be placed in the 4th or 5th position.

   (c) Professor Mcgonnagal's obstacle cannot be adjacent to Professor Quirrel's obstacle.

   How many possible orders were there for the obstacles?

3. A composite number $n$ is known to be super magical if the sum of its digits is three times its largest prime factor. What is the least three-digit super magical number?

# 7 Live Round: Set 7

1. Gilbert the Gecko and Octavius the Owl live on (1, 3) and (2, 2) of the coordinate plane, respectively. Their house is part of a square neighborhood with vertices (0,0), (4, 0), (4, 4), and (0, 4). Everyday, each of them travel either up, down, left, or right randomly and with equal probability by 1 unit. The probability that, after at most 3 moves, only 1 of the two leave their neighborhood (goes outside the square), while the other does not can be written as $\frac{m}{n}$, where m and n are relatively prime integers. Find $10m + n$. Note: Once someone leaves the neighborhood, they do not move anymore. Being on the border is also leaving the neighborhood.

2. Square $TONK$ and triangle $RON$ are located in planes that are perpendicular to each other. Given that $RO = 6, RN = 8$, and $ON = 10$, the length of $RK$ can be written as $m\sqrt{n}$. Find m+n.

3. Find the value of $\frac{x}{y}$ given the two following equations:

   (a) $(\log_{16} x) + (\log_8 y^3) = 6$

   (b) $(\log_{16} y) + (\log_8 x^3) = 9$

# 8 Live Round: Set 8

1. Ron has three galleons in his pocket. Two of the galleons are fair, and one galleon is double-headed. He picks a galleon at random and flips it 6 times. It comes up heads each time. The probability that he picked the galleon that was double headed can be written as $\frac{m}{n}$, where m and n are relatively prime integers. Find $m + n$.

2. $f(x) = \frac{4^x}{25^x} + \frac{5^x}{2^x}$. The value of $f(\frac{1}{1-\log_{10} 4})$ can be written as $\frac{m}{n}$, where m and n are relatively prime integers. Find $m + n$.

3. Point $P$ is outside a circle of unknown radius. A tangent is drawn from point $P$ which intersects the circle at point $A$. A second line is drawn from point $P$ which intersects the circle in two points, $B$ and $C$, respectively. Points $A$ and $C$ form the diameter of the circle. If $PA = 2$ and $PB = \sqrt{2}$, the area of the circle outside the shaded region (triangle $PAC$ is shaded) can be written as $\frac{a\pi}{b} - \frac{c}{d}$, where $a$ and $b$ are relatively prime integers and $c$ and $d$ are relatively prime integers. Find $a + b + c + d$.